Resolution Enhancement of Biomagnetic Images Using the Method of Alternating Protections

Seho Oh, Ceon Ramon, *Member, IEEE,* Robert J. Marks II, *Senior Member, IEEE,* Alan C. Nelson, and Michael G. Meyer

Abstract— **Resolution of biomagnetic images using the technique of the alternating projections is proposed. Our image reconstruction procedure is divided in two steps. First, the biomagnetic inverse problem is solved by use of the projection theorem to reconstruct an initial image of the current distribution from a given magnetic field profile. Although the current distribution thus obtained has poor resolution, it can resemble the original shape of the current distribution. The second step improves the resolution of the reconstructed image by using the method of alternating projections. The procedure assumes that images can be represented by line like elements and involves finding the line like elements based on the initial image and projecting back onto the original solution space. Simulation studies were performed on a set of parallel conductors and a shape of the conductors in the form of letters, UWB@. All conductors were of line like thickness. Restored images closely resemble the original shape of the conductors.**

I. Introduction

BIOMAGNETIC imaging concerns reconstruction of a current distribution from its measured magnetic field. In a previous paper [1], we have proposed use of a pseudo-inverse technique for doing so. The reader is referred to this paper for motivation and citation of previous work regarding this important problem. In this paper, the reconstruction is restored by the use of the method of alternating projections, a technique commonly used in signal recovery and synthesis. A commonly used special case of alternating projections is POCS, an acronym for *projection onto convex sets* [2]. Although alternating projections have been applied to various types of tomography [2], [3], this is the first time, to our knowledge, it has been applied to the biomagnetic image reconstruction.

II. Alternating Projections and POCS

The method of *alternating projections* is a powerful image restoration technique. It allows synthesis or reconstruction of images that satisfy two or more constraints. The performance of alternating projections can be concisely described when the constraint sets are convex. The alternating projection paradigm is then referred to as POCS for *projection onto convex sets.*

Let $\mathbb{C}$ denote a constraint set of functions, $\{\phi\}$, in a single space. A constraint set is convex if $\phi_1 \in \mathbb{C}$ and $\phi_2 \in \mathbb{C}$, implies that, for any α in the interval $0 \leq \alpha \leq 1$, the kernal $\alpha\phi_1 + (1-\alpha)\phi_2 \in \mathbb{C}$. Thus, a set is convex if, for every two points chosen within the set, all of the points in the line segment connecting the two points are also in the set. Geometrical shapes corresponding to convex sets include balls, line segments, planes, boxes, and quadrants.

The convexity of the constraint sets allows use of the powerful synthesis procedure of *projection onto convex sets* (POCS). POCS was initially introduced by Bregman [4] and Gubin *et al.* [5], and was later popularized by Youla and Webb [6], and Sezan and Stark [7] and has been applied to such topics as sampling theory [8], fuzzy set theory [9], and artificial neural networks [10], [11]. A superb overview of POCS with other applications is in a book edited by Stark [2].

A. Convex Set Projections

The *projection* of an arbitrary function Z, onto a (compact) convex set $\mathbb{C}$ is the unique function in $\mathbb{C}$ that is closest to Z in the mean square sense. Denote the projection operator by $\wp_{\mathbb{C}}$. Note that, if $Z \in \mathbb{C}$, then $\wp_{\mathbb{C}} Z = Z$. Projection operators are thus indempotent. If a function is already within the set, then the projection is an identity operation. It follows that $\wp^2_{\mathbb{C}} = \wp_{\mathbb{C}}$.

A list of useful projection operators can be found in Youla and Webb's paper [6] and in Stark's book [2].

B. Alternating Projections

There are three fundamental lemmas in the theory of POCS.

Lemma 1: Alternately projecting between two or more convex sets with a nonempty intersection will iteratively converge to a point common to all sets [2],

Note that the point of convergence generally depends on the initialization. If, however, there is a single point of intersection, e.g., two lines, then convergence will be independent of the initialization.

Lemma 2: Alternately projecting between two intersecting convex sets will converge to a limit cycle between points in each set closest to the other set [12].

This property can be used to find the best member in a set that is closest to another set in the mean square sense. Note

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S. Oh, R. J. Marks II, and M. G. Meyer are with the Department of Electrical Engineering, University of Washington, Seattle, WA 98195.

C. Ramon is with the Center for Bioengineering and the Center for Image Systems Optimization, University of Washington, Seattle, WA 98195.

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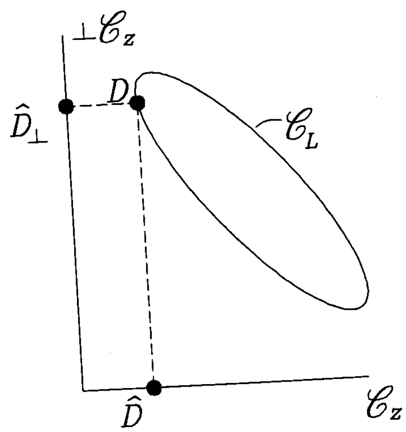

Fig. 1. The known vector image, $\hat{D}$, is the projection of the true object, D, onto the column space, $\mathbb{C}_z$ of the matrix R_z. The true object, D, is also known to lie in a set of line like objects, $\mathbb{C}_L$.

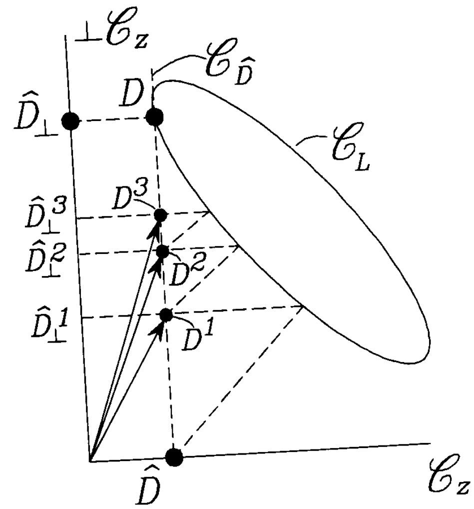

Fig. 2. Illustration of how altering projections can be used to reconstruct the object, D.

that, as can be visualized in the case of two parallel line convex sets, the limit cycle is not unique.

This property generalizes to more than three sets in the following sense. Let two or more constraint sets have a nonempty intersection, $\mathbb{C}_a$. Let two or more other constraint sets have a nonempty intersection, $\mathbb{C}_b$. If $\mathbb{C}_a$ and $\mathbb{C}_b$ do not intersect, then POCS will converge to a limit cycle between points convex set $\mathbb{C}_a$ and $\mathbb{C}_b$ each closest to the other in the mean square sense.

Lemma 3: Alternately projecting between three or more nonintersecting convex sets will result in a limit cycle that can be dependent on both the ordering of the projections and the initialization [13].

This final lemma states, unfortunately, that POCS can yield results of questionable worth when three or more of the convex sets do not intersect. The method of alternating projections can work even the underlying constraint sets are not convex [2].

III. Application to Biomagnetic Computed Tomography

We consider use of generalized projections to biomagnetic computed tomography of line like objects. The results illustrate the potential use of alternating projection techniques to biomagnetic computed tomography.

Let $[D]$ denote the magnitude of the current flow on the plane whereon current distribution construction is being performed (i.e., the (x, y) plane in [1]). The corresponding measurement of the magnetic field, $[B]$, at a distance z is given by the matrix equation

$$[B] = [R_z][D] \tag{1}$$

where ijth component of $[R_z]$ is

$$\frac{\mu_0 r_{ij,z}}{4\pi|\underline{r}_i - \underline{r}_j'|^3}$$

and $r_{ij,z}$ is the z component of $(\underline{r}_i - \underline{r}_j')$. We are using notation consistent with that of Ramon *et al.* [1]. Since the matrix $[R_z]$ is not full column rank, direct inversion of the observation $[B]$ to the source $[D]$ is not possible. We, rather, settle for the projection of $[B]$ onto the column space of $[R_z]$ using a projection

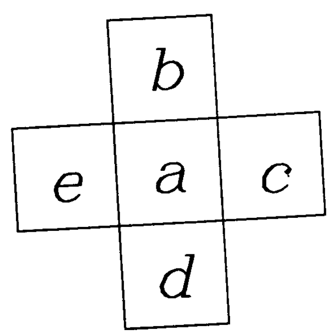

Fig. 3. Five pixels in an image. The line like projection is obtained by comparing pixel a to the surrounding pixels.

$$[D] \approx [\hat{D}] = [R_z]^T([R_z][R_z]^T)^{-1}[B]. \tag{3}$$

The image $[\hat{D}]$ is the closest to $[D]$ that lies in the column space of $[R_z]$. (The column space of $[R_z]$ is the set of all vectors that can be represented as a linear combination of the columns of $[R_z]$.) Note that this operation is a projection onto a convex set.

We can equivalently write (3) as

$$[\hat{D}] = [P_z][D] \tag{4}$$

where

$$[P] = \wp_z = [R_z]^T([R_z][R_z]^T)^{-1}[R_z] \tag{5}$$

is the matrix that projects onto the column space, $\mathbb{C}_Z$, of the matrix, $[R_z]$. This is illustrated in Fig. 1. It remains to find the vector $[\hat{D}_\perp]$ so that

$$[D] = [\hat{D}] + [\hat{D}_\perp]. \tag{6}$$

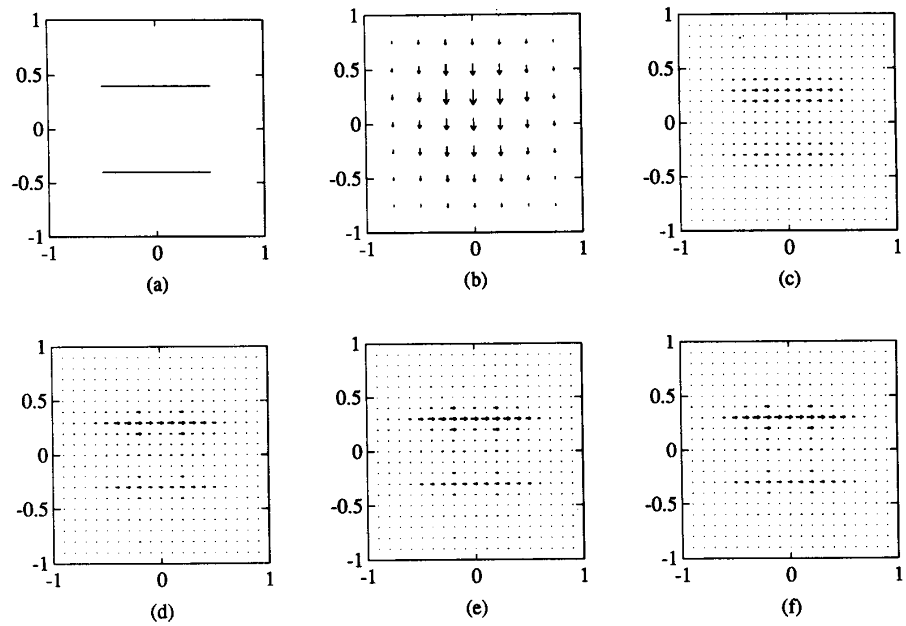

Fig. 4. Reconstruction of two parallel conductors. (a) The geometry of the conductor, (b) vector plot of x and y component of magnetic field, (c) reconstructions based on the minimum norm solution. Image restorations based on alternating projections: (d) first iteration, (e) eighth iteration, (f) sixteenth iteration.

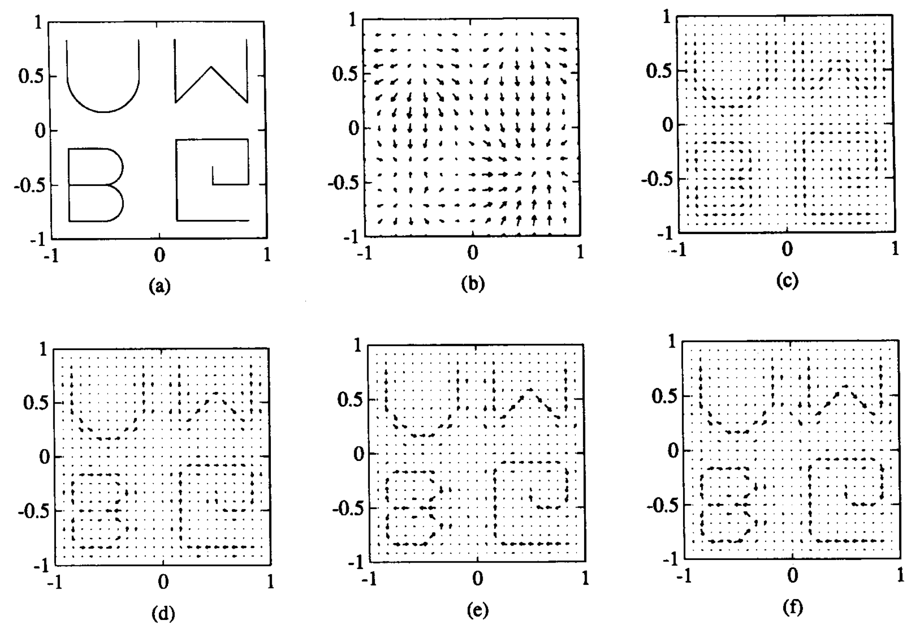

Fig. 5. Reconstruction of UWB@ letter shaped conductors. (a) The geometry of the conductor, (b) vector plot of x and y component of magnetic field, (c) reconstructions based on the minimum norm solution. Image restorations based on alternating projections: (d) first iteration, (e) eighth iteration, (f) sixteenth iteration.

The vector $[\hat{D}_\perp]$ lies in the orthogonal complement of the $\mathbb{C}_Z$ space. This is the space of all vectors that are orthogonal to each row of $[R_z]$. Denote this space by $\perp\mathbb{C}_Z$.

To estimate $[\hat{D}_\perp]$, additional constraints must be imposed on the restored object. We here illustrate this by requiring that the reconstructed image be "line like." In Fig. 1, the set of image vectors that satisfy the line like constraint set denoted by $\mathbb{C}_L$. Note that the desired reconstruction, $[D]$, lies in this set. Denote the projection operator into this space by $\wp_L$.

A geometrical illustration of the iterative alternating projection restoration is illustrated in Fig. 2. We first $[\hat{D}]$ onto the space of line like image vectors, $\mathbb{C}_L$. This is next projected

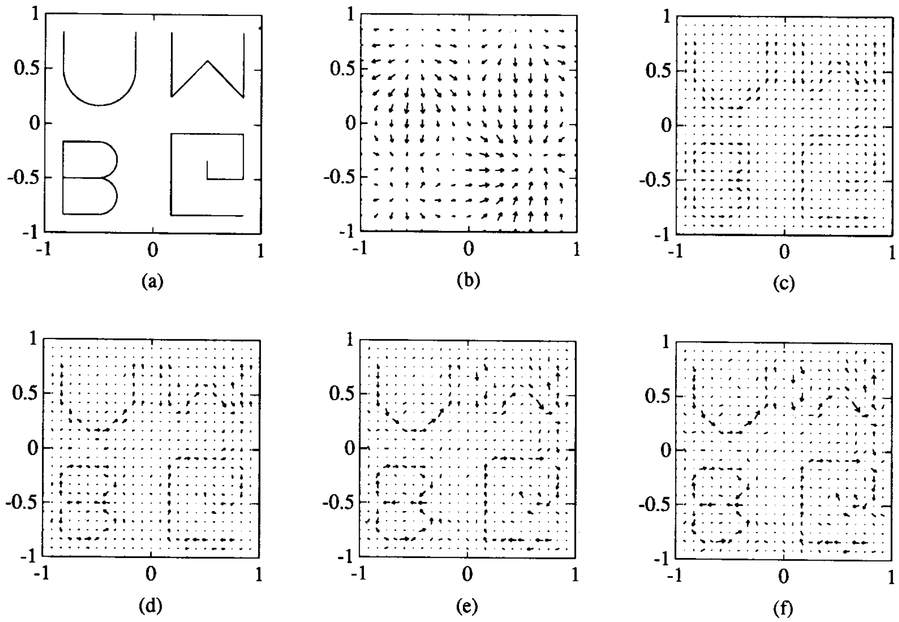

Fig. 6. Reconstruction same as Fig. 6 with additive noise. The noise levels 40 dB with respect to maximum amplitude of magnetic field.

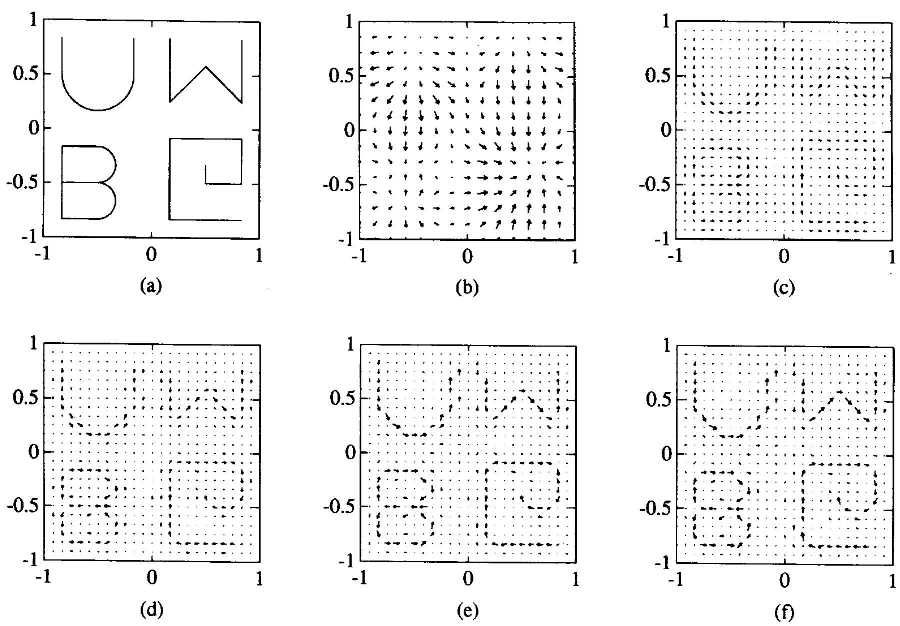

Fig. 7. Reconstruction same as Fig. 6 with additive noise. The noise level is 46 dB with respect to maximum amplitude of magnetic field.

onto the orthogonal complement of the column space of $[R_z]$ resulting in $[\hat{D}_\perp^1]$ using the operator

$$\wp_z^\perp = \mathbb{I} - \wp_z = [I] - [P].$$

where $\mathbb{I}$ is the identity operator and $[I]$ is an identity matrix. This vector is added to $[\hat{D}]$ to give the revised image estimate, $[D^1]$. The iteration is repeated. The image vector, D^1 is projected onto the space of line like objects. This vector is then projected onto the $\perp\mathbb{C}_Z$ space using $\wp_z^\perp$, and is added to $[\hat{D}]$ to result in the revised estimate, $[D^2]$. The iteration is repeated to yield $[D^3]$, etc. Assuming the translated column space (linear variety set) and line like constraint set intersect only at the desired reconstruction point then, as $n \rightarrow \infty$, we expect $[\hat{D}_\perp^n] \rightarrow [\hat{D}_\perp]$ and $[\hat{D}^n] \rightarrow [D]$.

Note that the iteration shown in Fig. 2 is simply alternating between the line like set of vector images, $\mathbb{C}_L$, and the set of all vectors whose projection onto $\mathbb{C}_Z$ is $[\hat{D}]$. Denote this set of vector images by $\mathbb{C}_{\hat{D}}$. This set is a plane translated from the origin.

There exist numerous methods for projecting onto line like objects. We chose the following. With reference to Fig. 3, we need to decide whether pixel, a, is from a line like surface. If the answer is "no," we set it to zero. Pixel a is set to zero if

$$(e \ \& \ b) > a \text{ or } (b \ \& \ c) > a \text{ or } (c \ \& \ d) > a \text{ or } (d \ \& \ e) > a.$$

Otherwise, the pixel is left as is. The notation $(e \ \& \ b) > a$ means that both e and b exceed a. The corresponding operator, $\wp_L$, does not correspond to a convex set projection. It is, however, indempotent, in the $\wp_L^2 = \wp_L$.

IV. Eamples of Reconstructions Based on Minimum Norm Solutions and Resolution Enhancement

The geometry of a parallel conductor and the $x - y$ component of the magnetic fields are shown in Fig. 4(a) and (b). The magnetic field is sampled at the height of 0.4 units above the plane containing the conductor. The number of grid partitions used in the reconstruction were 21×21. The parallel conductor shape is visible in the vector intensity plot. Iterative improvements are shown in Fig. 4(d)–(f). Increasing the number of iterations from one to sixteen reduces the width of the conductor and peaks in the reconstruction become sharper.

The second example considered is a combination of several conductor geometries in the shape of letters UWB@ located in a plane of size 2×2 units. The geometry of the conductors is shown in Fig. 5(a). The magnitude of the current flowing in each conductor is 1 A. The $x - y$ component of the magnetic field at the height of 0.25 units is shown in Fig. 5(b). 15×15 samples of the magnetic field in a space of 2×2 units were taken. Notice the surface of the magnetic field shows several peaks and valleys but it does not reveal the shape of the conductors. The vector intensity plot of reconstruction based on minimum norm solution is given in Fig. 5(c). The shape of the conductors are barely recognizable in this figure. Iterative improvements are given in Fig. 5(d)–(f). Reconstruction based on minimum norm solution, Fig. 5(c), shows all conductors are approximately 0.2 units wide. The original width of the conductors is recovered and resolution is improved in going from first to sixteenth iterative reconstructions. However, some of the current elements are slightly displaced with respect to the original definition of the conductors.

The third and fourth examples are the same as the second example with the noise in the magnetic field. The noise is identically independent gaussian noise with the 40 dB (Fig. 6) and 46 dB (Fig. 7) with respect to the maximum amplitude of the all magnetic field [1]. In Fig. 6, the reconstruction is visibly degraded.

In all the above examples we have shown that the reconstruction based on a minimum norm solution does provide the recognizable shape of the conductors but with poor resolution. Conductors tend to have width larger than the original shape in the reconstructions based on minimum norm solution. The resolution and the width of reconstruction can be improved by some iterative procedures. To this point, the performance of the iterative method is highly dependent on the accuracy of the minimum norm solution, especially if the magnetic field is corrupted by noise.

V. Dicussion and Conclusions

We have shown that alternating projections can be used to significantly enhance the resolution of reconstructed biomagnetic images. The reconstruction consists of generation of the pseudo-inverse of Biot–Savart's law from the sampled magnetic field. The restoration is then iteratively performed by alternatingly projecting between two or more constraint sets and the pseudo-inverse. The procedure was illustrated using a line like constraint set.

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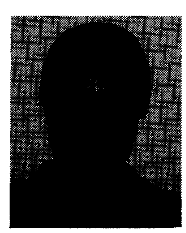
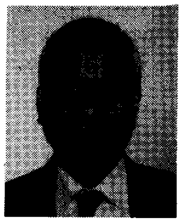


Seho Oh received the B.S. degree in electronics engineering from Seoul National University and the M.S. degree in electrical engineering from Korea Advanced Institute of Science and Technology, Seoul.

From 1981 through 1986, he was in Central Research Laboratory of Goldstar Company. He received the Ph.D. degree in electrical engineering from the University of Washington, Seattle, in 1989. His research interests are in the area of signal analysis, artificial neural networks, pattern recognition and biomagnetic imagings. He is the co-author of over 40 archival and proceedings papers and has been issued two United States patents.



Ceon Ramon (M'81) is a Research Assistant Professor in the Center for Bioengineering at the University of Washington where he is involved in establishing a biomagnetic imaging program. He received the Ph.D. degree in electrical engineering from the University of Utah, Salt Lake City, in 1973. He has been active in biomedical research for the last twenty years. His current research interests are in biomagnetism, bioelectromagnetics and imaging.

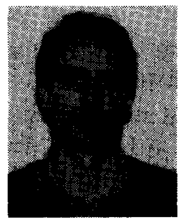
Dr. Ramon is a member of Sigma Xi, Etta Kappa Nu, Bioelectromagnetics Society, and IEEE.

Robert J. Marks II (S'71–M'72–SM'83) is a Professor in the Department of Electrical Engineering at the University of Washington, Seattle. He was awarded the *Outstanding Branch Councilor* award in 1982 by IEEE and, in 1984, was presented with an *IEEE Centennial Medal.* He was named a Distinguished Alumnus of Rose-Hulman Institute of Technology in 1992. He was Chair of *IEEE Neural Networks Committee* and was the co-founder and first Chair of the *IEEE Circuits & Systems Society Technical Committee on Neural Systems & Applications.* He served as the first President of the *IEEE Neural Networks Council* (1990–91). He was named an *IEEE Distinguished Lecturer* in 1992. He is a Fellow of the *Optical Society of America.* He was the co-founder and first President of the *Puget Sound Section of the Optical Society of America* and was elected that organization's first *Honorary Member.* He is co-founder and current President of *Multidimensional Systems Corporation* in Lynnwood, WA.

Dr. Marks is the Editor-in-Chief of the IEEE TRANSACTIONS ON NEURAL NETWORKS (1992-present). He was the topical editor for *Optical Signal Processing and Image Science* for the *Journal of the Optical Society on America - A* (1989–91) and a member of the Editorial Board for *The International Journal of Neurocomputing* (1989–92). Dr. Marks served as *North American Liaison* for the 1991 Singapore *International Joint Conference on Neural Networks* (IJCNN), *International Chair* of the 1992 *RNNS/IEEE Symposium on Neuroinformatics and Neurocomputing* (Rostov-on-Don, USSR) and Organizational Chair for both the 1993 IEEE Virtual Reality Annual International Symposium (VRAIS) in Seattle and the *IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis* (Victoria, BC, 1992). He also served as the *Program* and *Tutorials* Chair for the *First International Forum on Applications of Neural Networks to Power Systems* (Seattle, WA, 1991) and the General Chair of the *International Symposium on Circuits and Systems* (Seattle, WA, 1995). Dr. Marks has two U.S. patents in the field of artificial neural networks. He is author of the books *Introduction to Shannon Sampling and Interpolation Theory* (Springer-Verlag, 1991) and *Advanced Topics in Shannon Sampling and Interpolation Theory* (Springer-Verlag, 1993). Dr. Marks is a co-founder of the Christian Faculty Fellowship at the University of Washington and serves as the faculty advisor to the University of Washington's chapter of *Campus Crusade for Christ.*

Alan C. Nelson received the Ph.D. degree in biophysics from the University of California, Berkeley, in 1980.

He became an associate Professor in Nuclear Engineering at MIT where he was instrumental in developing the Radiological Science graduate program, subsequently, he joined the Center for Bioengineering at the University of Washington where he formed the Center for Imaging Systems Optimization and developed the graduate program in Biomedical Imaging. He founded a biomedical imaging company, NeoPath, Inc., where he serves as Chairman of the Board. His research interests focus on biomedical imaging hardware, processing algorithms, and reconstruction techniques.



Michael G. Meyer began his career at IBM Corporation after obtaining his BSEE degree from the University of Illinois-Urbana. His recent work at the University of Washington focused on defining techniques to be used in biomagnetic imaging and culminated in a MSEE degree. He is currently on staff at NeoPath, Inc., where his work centers on developing algorithms for automated screening of PAP smear slides for cancer detection.