SYMBOLS UNITS NOMENCLATURE

This department is intended to focus attention on those Letters in which the principal consideration is improved terminology: technical expression rather than technical content. They will be refereed in the same manner as Letters, but by different criteria. SUN Letters should be submitted to the Editor, APPLIED OPTICS, 7 Norman Road, Newton Highlands, Mass. 02161. Readers are invited to comment on this experimental SUN department.

Line Spread Function Notation

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In the process of characterizing a linear system, the system line spread function (impulse response) is normally written in the form[1–2]

$$h_1(x_o; \xi) = S[\delta(x_i - \xi)], \tag{1}$$

where $S[\,\cdot\,]$ is the system operator, $\delta(x)$ is the Dirac delta, and x_o and x_i are, respectively, the output and input variables. Upon assumption of space-invariance (isoplanicity), one writes

$$h_1(x_o; \xi) = h(x_o - \xi). \tag{2}$$

That is, the line spread function shifts directly with the input impulse and thus depends only on the coordinate difference $x_o - \xi$.

At least five authors[3–7] have utilized the less widely used line spread function notation

$$\begin{aligned} h(x_o - \xi; \xi) &= S[\delta(x_i - \xi)] \\ &= h_1(x_o; \xi). \end{aligned} \tag{3}$$

The most obvious advantage of this notation, as noted by Lohmann and Paris,[3] is the cleaner transition to the invariant case. The line spread function merely becomes independent of its second argument:

$$h(x_o - \xi; \xi) = h(x_o - \xi). \tag{4}$$

Note that the function $h(x_o - \xi)$ here is equivalent to that in Eq. (2).

A second advantage of the amended line spread function notation is the straightforward manner in which one can express a space-variant system's transfer function. Consider first the procedure in the isoplanatic case. One probes the input with a single-shifted impulse, $\delta(x_i - \xi)$, finds the corresponding output, $h(x_o - \xi)$, shifts this output to obtain $h(x_o)$, and last performs a Fourier transform to arrive at the system transfer function. One may perform an analogous procedure using the amended line spread function notation to arrive at the transfer function of a space-variant system:

$$H_x(f_x; \xi) = \mathcal{F}_{x_o}[h(x_o; \xi)], \tag{5}$$

where f_x is the spatial frequency domain variable and $\mathcal{F}_{x_o}[\,\cdot\,]$ is the Fourier transform operator in x_o defined by

$$\mathcal{F}_{x_o}[h(x_o; \xi)] = \int_{-\infty}^{\infty} h(x_o; \xi) \exp(-j2\pi f_x x_o) dx_o\,. \tag{6}$$

Space-variant transfer functions of the form of Eq. (5) are employed in the piecewise isoplanatic approximation treatment of variant systems[6] as well as in developing a sampling theorem for space-variant systems.[7] Expressions of this form employing the conventional line spread function notation of Eq. (1) are less intuitive. Other difficulties arising when using the conventional notation are discussed by Kailath.[4]

The amended line spread function lends itself nicely to Fourier transformation with respect to its variation variable ξ. Such a computation, for example, is necessary in applications of the space-variant system sampling theorem.[7] One looks at

$$\mathcal{F}_\xi[h(x_o; \xi) = \int_{-\infty}^{\infty} h(x_o; \xi) \exp(-j2\pi \upsilon \xi) d\xi. \tag{7}$$

If this expression is band limited in υ for all x_o (i.e., if it has a finite *variation bandwidth*), the sampling theorem may be applied. A second example of use is the direct computation of a linear system's output spectrum $G(f_x)$ corresponding to an input $f(\xi)$[7]:

$$\begin{aligned} G(f_x) &= \mathcal{F}_{x_o}[g(x_o)] \\ &= \mathcal{F}_{x_o}\left[\int_{-\infty}^{\infty} f(\xi) h(x_o - \xi; \xi) d\xi\right] \\ &= \int_{-\infty}^{\infty} f(\xi) \mathcal{F}_{x_o}[h(x_o; \xi)] \exp(-j2\pi f_x \xi) d\xi \\ &= \mathcal{F}_\xi\{f(\xi)\mathcal{F}_{x_o}[h(x_o; \xi)]\}|_{\upsilon = f_x}\,, \end{aligned} \tag{8}$$

where, in the second step, we have utilized the superposition integral statement of the output.[1] Employing Eq. (5) we may rewrite Eq. (8) as

$$G(f_x) = \mathcal{F}_\xi[f(\xi) H_x(f_x; \xi)]|_{\upsilon = f_x}\,. \tag{9}$$

Note again the clean transition to the isoplanatic case when Eq. (9) takes on the familiar product form

$$G(f_x) = F(f_x) H_x(f_x), \tag{10}$$

where $F(f_x)$ is the Fourier transform of the input.

The only disadvantage to use of the amended line spread function notation is that it takes longer to write. In view of the stated advantages, however, this would appear to be of minor concern.

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References

1. J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).
2. J. B. Thomas, *An Introduction to Statistical Communication Theory* (Wiley, New York, 1969).
3. A. W. Lohmann and D. P. Paris, J. Opt. Soc. Am. **55**, 1007 (1965).
4. T. Kailath in *Lectures on Communication System Theory,* E. J. Baghdady, Ed. (McGraw-Hill, New York, 1960).
5. N. Liskov, "Analytical Techniques for Linear Time-Varying Systems," Ph.D. thesis, Cornell U., 1964.
6. R. J. Marks II and T. F. Krile, accepted for publication in Appl. Opt. (1976).
7. R. J. Marks II, J. F. Walkup, and M. O. Hagler, J. Opt. Soc. Am. **66** (Sept. 1976).

Meeting Reports

Information about future meetings should be sent to
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Society for Information Display, International Symposium, Beverly Hills, 4–6 May 1976

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The 1976 SID International Symposium held this year in Beverly Hills, California, had the interdisciplinary and occasionally cross-cultural flavor that might be expected from an association of fundamental science, technology, and entrepreneurship. In this meeting the fundamental side was research in vision and perception, devoted to outlining the capacities and characteristics of the visual system as a guide for display development. Since technology must ultimately either be successfully marketed or discarded, the entrepreneural viewpoint was of real interest. However, the bulk of the papers were devoted to research on the various current or developing display technologies. Multiple sessions and a finite attention span kept this reviewer from attending everything, so this report must be only what, to his interests, represented highlights.

Image evaluation factors were the subject of isolated papers in various sessions as well as in one full session and an informal discussion panel. The later, chaired by **Sol Sherr** and comprising **W. L. Carel, R. W. Cohen, D. A. Shurtleff, H. G. Slottow,** and **H. L. Snyder,** gave an overview of this field. Two distinct approaches seem evident: first, efforts to develop more adequate descriptions of the quality of images as dealt with by the human perception system; and second, operational measurements of the effects of various technical characteristics, such as type fonts, use of color, etc., on specific recognition and observer response tasks.

The Keynote paper by **James Hillier** outlined the business problems encountered when introducing communications products that require many users to make a viable system. Display technologies addressing such markets can experience a discouraging period of slow growth until a threshold penetration of the market is reached. On a related theme, the banquet address of **Henry Kloss** described the chronology of problem solving required to create and market his company's projection television system. In an invited paper in the keynote session, **Albert Rose** discussed the demands that available light photography place on the camera. He concluded that electronic cameras must have quantum efficiencies near 100% to reproduce with fidelity scenes at typical indoor light levels for TV display. CCD cameras may have this potential.

The session on liquid crystal devices was highlighted by two multielement matrix displays: a 0.4 × 0.5-m 400 × 500 spot multiplexed panel presented by **K. Ono, E. Mitani, E. Kaneko,** and **M. Sato,** and a 2 in. × 2 in., 200 × 200 element MOS-addressed TV display reported by **C. P. Stephens** and **L. T. Lipton.** Both systems used dynamic scattering. A 1 in. × 1 in. version of the latter display was shown at the conference and produced real-time TV of impressive quality.

Other liquid crystal topics presented at the session included bar graph displays (**W. L. Carel** and **C. R. Stein, S. Sherr**), twisted nematic multiplexing (**A. R. Kmetz** *et al.*), and high speed addressing techniques for field effect devices (**D. J. Channin**). In another session, **J. Polleck** and **J. Flannery** described a new electrohydrodynamic texture and its application to photoconductive image amplification devices. Another paper on liquid crystal imaging devices by **J. E. Adams** *et al.* emphasized the use of a blocking layer between the photoconductor and its electrode to enhance the device sensitivity.

In the session on matrix displays, two papers by **C. Suzuki** *et al.* described characteristics of electroluminescent panels with internal memory and optical writing capability. **B. Frescura** analyzed from circuit considerations the size limits of LED monolithic arrays, which indicate that a 2 in. × 2 in., 144 × 144 element display may be possible. **L. Lee** showed early results on a novel display using magnetic spheres, one side dark and one side light, which orient in magnetic fields. Such a display would have inherent memory and addressing characteristics similar to computer magnetic cores. Along similar lines, a light valve device using free-standing electret films was described by **J. L. Bruneel, J. J. Crosnier,** and **F. Micheron** in another session.

Numerous papers on plasma display panels reflected the advanced state of development of these systems in relation to other flat panel technologies. Of particular novelty were a green phosphor plasma display (**H. Yamashita, S. Andoh, T. Shinoda**), an internal addressing operation for simplified multiplexing (**T. Criscimagna** *et al.*), and a technique for driving a plasma display to invert its on–off state (**P. Ngo**).

The CRT remains the most advanced and widely used technology for image display and presentation of graphical or complex alphanumeric information. **J. Schwartz** reviewed the optical characteristics of commercial color TV picture tubes and discussed the potential for duplicating this performance in various flat panel technologies. Nothing at present stands out as a serious competitor. However, **R. Schulman** and Schwartz presented a paper in the same session on a flat panel cathodoluminescence system for TV display. In the device they described, an electron multiplier structure is combined with an ion feedback process to enhance the electron current to obtain high brightness.

As a whole, the conference nicely balanced the significant aspects of the information display field.