

Mini Quiz ELC5370
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We invest a dollar in the stock market. The first day the market goes up 2% so our investment becomes \$ 1.02. The next day the market goes down 3% so our stock is now worth $1.02 \times 0.97 = \$0.9894$.

Let $\{X_1, X_2, \dots, X_n\}$ denote the market fluctuations over n days. For the example above, $X_1 = 0.02$ and $X_2 = -0.03$. What is the value of your stock in n days if each X_i is i.i.d. on the interval $-0.05 \leq x \leq 0.05$? Assume n is large.

$$\begin{aligned} Y_n &= \prod_{i=1}^n (1 + X_i) \\ \frac{1}{n} \ln Y_n &= \frac{1}{n} \sum_{i=1}^n \ln(1 + X_i) \rightarrow E[\ln(1 + X)] \\ E[\ln(1 + X)] &= 10 \int_{-0.05}^{0.05} \ln(1 + x) dx \\ &= 10 \left[(1+x) \ln(1+x) - x \right] \Big|_{-0.05}^{0.05} \\ &= 10 \left\{ [(1.05) \ln(1.05) - 0.05] - (0.95 \ln 0.95 + 0.05) \right\} \\ &= 10 [0.05123 + (0.04873) - 0.1] = -0.0042 \end{aligned}$$

THUS: $\ln Y_n = -0.0042 \times n$
 $\implies Y_n = e^{-0.0042 \cdot n} = (0.999583)^n$

NOT
GOOD!