

Solution

Mini Quiz Professor Marks

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Evaluate

$$\int_0^{\infty} \text{sinc}(t) \text{sinc}\left(\frac{t}{2}\right) dt.$$

Note that the integrand is even.

$$\int_0^{\infty} \text{sinc}(t) \text{sinc}\left(\frac{t}{2}\right) dt = \frac{1}{2} \int_{-\infty}^{\infty} \text{sinc}(t) \text{sinc}\left(\frac{t}{2}\right) dt$$

Parseval's Theorem:

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \int_{-\infty}^{\infty} X(u) Y^*(u) du$$

$$\text{sinc}(t) \leftrightarrow \Pi(u)$$

$$\text{sinc}\left(\frac{t}{2}\right) \leftrightarrow 2\Pi(2u)$$

$$\Rightarrow \frac{1}{2} \int_{-\infty}^{\infty} \text{sinc}(t) \text{sinc}\left(\frac{t}{2}\right) dt$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} \Pi(u) \cdot 2\Pi(2u) du = \int_{-\infty}^{\infty} \Pi(2u) du = \frac{1}{2}$$

