SET CONSTRAINT DISCOVERY: MISSING SENSOR DATA RESTORATION USING AUTO-ASSOCIATIVE REGRESSION MACHINES

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ABSTRACT

A sensor array can generate interdependent readings among the sensors. If the dependence is sufficiently strong, the readings may contain redundancy to the degree that the readings from one or more lost sensors may be able to be accurately estimated from those remaining. An autoassociative regression machine can learn the data interrelationships through inspection of historical data. Once trained, the autoassociative machine can be used to restore one or more arbitrary lost sensors if the data dependency is sufficiently strong. Recovery techniques include *alternating projection onto convex sets* (POCS) and iterative search algorithms.

Key Words: POCS, neural network, auto-encoder, sensor restoration, autoassociative regression machine.

1. INTRODUCTION

Consider the case where a plurality of sensors produces readings cross associated in a possibly nonlinear manner. In certain scenarios, these readings may be related in such a way as to allow restoration of one or more lost readings from those remaining. In many important cases, the sensor readings are related. Consider the simple case, for example, when a plurality of temperature sensors are placed in close proximity at various locations in an open room. Let one or more of the sensors fail. If the sensor readings are sufficiently dependant. *Missing sensor data* (MISED) *restoration* can estimate the readings of the failed sensors by recognition or discovery of a constraint placed on the historical readings from the sensor bank. Empirical discovery of such constraints from a historical database and their use in failed sensor reading restoration is the focus of this paper.

Constraint imposition on data sets is commonly assigned to a human expert charged with data set modeling. From the physics of the data generation or other process limitations, the expert can heuristically impose constraints. Examples of constraints imposed on a sensor bank array include minimum phase, symmetry, band limited, strictly increasing and non-negativity constraints. Model imposition of constraints allows powerful representation of data generated by the sensor bank. In contrast to model (or expert) constraint declaration, we propose to let the sensor data discover its own constraints. In many important cases, empirical constraint discovery

1. Can be used on data bases where no modeled set constraint is obvious,
2. May discover new subtle but important data constraints and
3. Will generally result in more restrictive and accurate constraints than is the case with expert constraint imposition.

Data set constraints imposed either empirically or by expert allow data set element representation in a lower dimension. Doing so, in many important cases, lets values of failed sensors be accurately estimated from those sensor readings remaining.

A common characteristic of a constrained data set is the ability to represent the set using fewer degrees of freedom. As an example, bandlimited vectors have discrete time Fourier transforms that are identically zero over a spectral interval. This property can be used to reduce the degrees of freedom of a bandlimited vector's representation. The reduced degrees of freedom can be used to form a constraint bottleneck wherein constraint set members pass unaltered. The bottleneck for a bandlimited signal, for example, is a low pass filter. This bottleneck can, in many important instances, be used to interrelate sensor readings and, in certain cases, restore missing readings from those available.

2. ALTERNATE APPROACHES TO MISSING SENSOR RESTORATION

State estimation is commonly used to identify state variables that are not accessible for direct measurements. The technique can be modified to the estimate missing sensor values. These model-based approaches have been investigated in various forms over three decades [19].

To estimate the data of the missing sensor, the state estimation method is configured to identify specified systems states using the information from the healthy sensors only. The known states are used to estimate the data of the missing sensors through direct query of the system model. This process is not robust and the solution can be unfeasible for real system applications. Often human intervention is used to interpret and subsequently modify or reject the state estimator data.

There exist related methods of estimating readings of missing sensors that deviate from the problem we consider. Adapting system performance in the presence of failed sensors [8,24], for example, requires knowledge of the desired system response. Variations of this approach include the Papoulis-Gerchberg, Gerchberg-Saxton, Fienup and related algorithms [6, 16, 19, 22].

4. DATA DEPENDANCY

Data dependency can be exploited to restore failed sensor readings. In the example of the closely spaced sensors, temperature readings will vary smoothly and readings from adjacent sensors can be used to restore those from failed systems. When the sensors outputs are related, missing values can possibly be restored as a function of those that remain operational as, for example, missing signal samples from a bandlimited signal [10, 18, 20, 21] or an image [5, 22]. Armitage & Lo [2], building on the work of Ahmad & Tresp [1], recognized the robustness of neural network performance is potentially due to the interrelationship among the training data. They propose use of missing value indicators to estimate missing sensors rather than placing the missing sensor values directly to zero. Conventional Bayesian analysis [1,28] and clustering [27] can also be applied.

Let the bank sensor values be $\{s_n \mid 1 \leq n \leq N\}$ and be represented in the vector $\vec{s}$. We propose determination of the relationship among sensor outputs using a sensor database applied to an autoassociative regression machine with input/output relation, $\vec{\sigma} = \vec{f}(\vec{s})$, trained using supervised learning. Rather than provide values only sufficient to maintain proper plant control, we propose to estimate the actual failed sensor data reading. The approach is applicable when target control performance is unknown or otherwise not available.

Motivated by the similar representation in vector quantization, the unity mapping is dubbed *an auto-encoder* [36]. This is illustrated in Figure 1. The resulting regression machine provides an identity mapping for the sensor database. Specifically, let there be P sets of sensor training data, $\{\vec{t}^{\,p} \mid 1 \leq p \leq P\}$. The auto associative regression machine is trained to produce an identity mapping for the training data. Specifically, we train $\vec{f}(\cdot)$ so that $\vec{f}(\vec{t}^{\,p}) = \vec{t}^{\,p}; 1 \leq p \leq P$. In doing so, the data is run through a bottleneck wherein the degrees of freedom are less than that of the data dimension. If the input can be reconstructed from the reduced data set, an empirical constraint of the data set has been discovered. Doing so is the first step in MISED restoration.

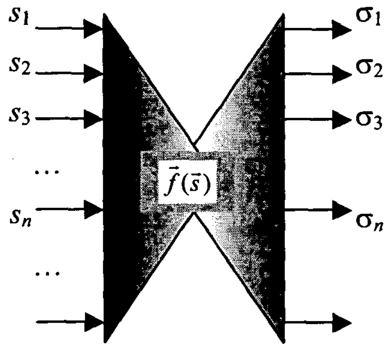

Figure 1: An auto-encoder produces a mapping $\vec{\sigma} = \vec{f}(\vec{s})$ that reduces to an identity, $\vec{s} = \vec{f}(\vec{s})$, when data from the sensor database is input.

A commonly used method of discovering constraints is singular value decomposition. The well-known auto-encoder neural network [3, 36] illustrated in Figure 2 can perform a commensurate operation. A neural network is trained to reproduce the input training data at the output. Since the process is run through a bottleneck in the hidden layer containing fewer than the number of inputs, the numbers of degrees of freedom has been decreased.

Although the encoder in Figure 2 uses sigmoid nonlinearties, the results are often commensurate with those obtained using a totally linear processor [15]. In a strictly linear model, the encoder in Figure 2 learns to perform a projection operation onto a subspace spanned by the training data.

As is illustrated in Figure 3, multiplying an arbitrary vector by the projection matrix results in a vector on the linear manifold (*a.k.a.* subspace) formed by the span of the data in the training data set [14]. The trained encoder in Figure 2 is more robust than results from direct computation of the projection matrix[1]. If, for example, one of the points in Figure 3 is moved slightly off the manifold (line), the data matrix becomes full rank and the projection matrix becomes a useless identity matrix. The encoder in Figure 2, on the other hand, identifies the best

[1] Denote the library of training data by the $N \times P$ data library matrix as a concatenation of all of the training data vectors as $\mathbf{T} = \left[\vec{t}^{\,1} : \vec{t}^{\,2} : \vec{t}^{\,3} : \cdots : \vec{t}^{\,p} \cdots : \vec{t}^{\,P}\right]$. The projection matrix onto the linear subspace spanned by the data is $\mathbf{P} = \mathbf{T}(\mathbf{T}'\,\mathbf{T})^{-1}\mathbf{T}'$ [17].

manifold when training vectors lie slightly off the defining manifold.

Use of the projection theorem is a standard approach to estimate a set of values known to lie on a specified manifold [14]. Kohonen [12] proposes projection as an approach to associative memories. In Figure 2, the training data represents noiseless data. Given noisy data, $\vec{s}$, the best solution is the projection of the noisy data onto the manifold. This results in $\mathbf{P}\vec{s}$.

6. MISED RESTORATION: THE LINEAR CASE

When sensor values are restricted to a linear manifold, values of one or more missing sensor readings can be reconstructed from those remaining.

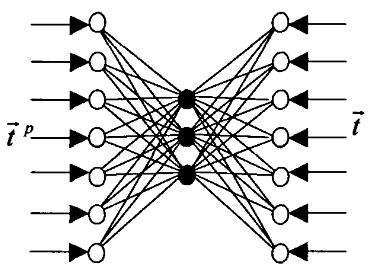

Figure 2: The auto-encoder neural network with a single hidden layer performs feature reduction in a manner similar to principal component analysis.

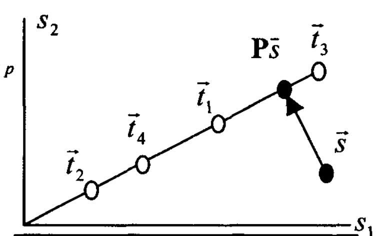

Figure 3: The projection matrix projects the arbitrary vector, $\vec{s}$, onto the linear manifold defined by the training data set.

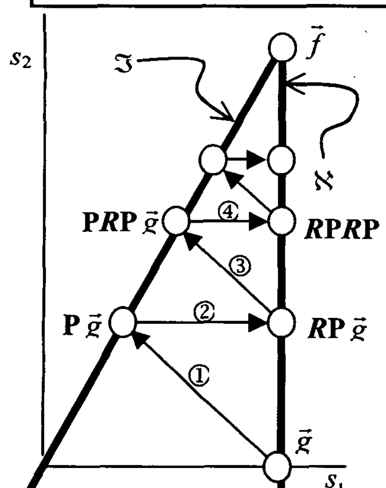

Figure 4: *Alternating projections onto convex sets* (POCS) illustrated for two intersecting planes. By alternatingly projecting between the subspace labeled $\Im$ and the plane labeled $\Re$, convergence is to a point common to the intersection of both.

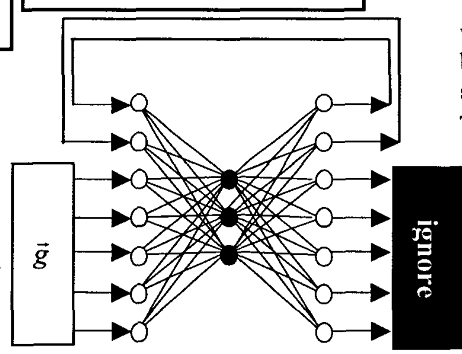

Figure 5: Using a trained autoassociative encoder to perform the POCS operation illustrated in Figure 5. The two top input nodes, on the left, correspond to two missing sensor values. The remainder of the input nodes are fed the values of the operational sensors recorded in the $\vec{g}$ vector. The output of the top two nodes are fed back to the input with unit gain. The known inputs continue to supply the values obtained from the working sensors. Through POCS theory, the top two nodes will converge, under general conditions, to the values that would have been recorded by the failed sensors.

6.1 POCS. One approach to restoration of the missing sensor reading is through the method of *alternation projection onto convex sets* (POCS). POCS has been applied to tomography [19], biomagnetic field reconstruction [25], signal analysis [26], image processing, radiography [13], holography [16], neural networks [38], and signal restoration [32].

For the failed sensor restoration problem, a two dimensional illustration of POCS is shown in Figure 4. The discussion to follow, however, is applicable in any dimension. There exists more complete developments and generalizations of POCS elsewhere [19]. Our purpose here is to provide a basic understanding of the performance of POCS and illustrate its potential usefulness in the MISED restoration.

The linear manifold in Figure 4 on which the training data lie is dubbed $\Im$. The vector containing sensor data to be restored is denoted by the vector $\vec{f}$ (for *find*). Clearly, the unknown $\vec{f} \in \Im$. The vector of sensor readings, with the failed sensor readings set to zero, is denoted by $\vec{g}$ (for *given*). The first step is to project the given vector onto the linear manifold defined by the training data. The result after step ①, illustrated in Figure 4, is $\mathbf{P}\,\vec{g}$. This vector, however, cannot be the restoration because the vector entries corresponding to working sensors do not contain the correct readings. Step ②, then, is to replace these vector components with the known correct values. The operation of replacing these vector components with the measured values while retaining whatever values are in the failed sensor location is denoted by $\boldsymbol{R}$. The set of all vectors containing the measured values of the working sensors with arbitrary entries corresponding to the failed sensors forms a linear variety (plane), $\aleph$, illustrated in Figure 4 by a vertical line. The operator, $\boldsymbol{R}$, projects onto the set $\aleph$. The result of step ②, after replacing the appropriate vector components to the correct values is, as shown in Figure 4,

$RP\vec{g}$. The projection is repeated in step ③ followed by insertion of the readings from the working sensors in step ④. The iteration is repeated. The final destination of the iteration is the intersection of $\Re$ and $\Im$ which is, of course, the vector containing correct values for the failed sensors, $\vec{f}$.

Reconstruction criteria can be established for two or more nonlinear convex sets[2]. Alternatingly projecting among two or more sets with nonzero intersection will result in convergence to a fixed point lying in the intersection of the sets[3]. Methods to accelerate convergence are available. For the problem illustrated in Figure 5, for example, three or more working sensors will generally suffice to restore the values that would be read from failed sensors. Convergence of POCS, as described in this example, is linear. For elaboration and details POCS properties, see Marks [19].

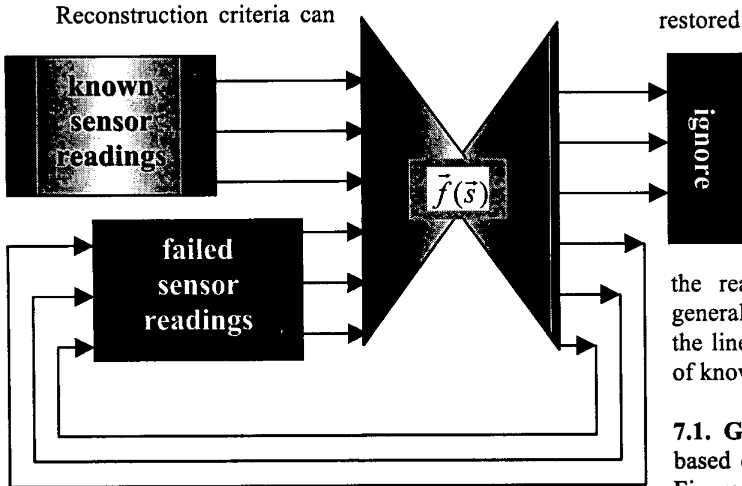

Figure 6: Illustration of restoring readings of failed sensors using a generalization of POCS. For an arbitrary set of failed sensors, the iteration will converge to the unique solution if the operation, $\vec{f}(\vec{s})$, is contractive. The feedback iteration can be regularized to increase the convergence rate.

7. GENERALIZED RESTORATION OF LOST SENSOR READINGS FROM DISCOVERED CONSTRAINTS

There will be data sets where the discovered constraints do not correspond to convex sets. In such cases, a POCS generalization approach or alternate search technique is more appropriate. The linear algorithm for restoration of readings of failed sensors can be extended to the nonlinear case. The steps are as follows.

1. Using training data, discover constraints among the readings of the sensors.
2. When sensors fails, restore them by requiring the restored vector of sensor values
   (a) adhere to the discovered constraint, and
   (b) agree with the readings from the operational sensors.

   In many important cases, the solution is unique.

There are two general methodologies we propose for restoring the readings of failed sensors. The first is a generalization of the POCS approach proposed for the linear case. The second is based on application of known optimization procedures.

7.1. Generalized POCS Approach: A procedure based on a generalization of POCS is illustrated in Figure 6. The encoder of Figure 1, trained from sensor data using supervised learning, performs the operation $\vec{f}(\vec{s})$. The readings from the operational sensors serve as a continuous input to the system. Values of the input corresponding to the failed sensors are initially given values of zero. The outputs corresponding to the lost sensor readings are fed to the input.

A useful criterion for unique and stable convergence of the iteration in Figure 6 rests on the concept of contractive and nonexpansive operators [1]. An operator, Θ, mapping $\Re^N \to \Re^N$, is *contractive* if, for all vectors $\vec{x}$ and $\vec{y}$, it follows that $\| \Theta\vec{x} - \Theta\vec{y} \| < \| \vec{x} - \vec{y} \|$, where $\|\cdot\|$ denotes the ℓ_2 norm. In other words, the vectors $\vec{x}$ and $\vec{y}$ are farther apart than the vectors $\Theta\vec{x}$ and $\Theta\vec{y}$. The operator is *nonexpansive* if $\| \Theta\vec{x} - \Theta\vec{y} \| \le \| \vec{x} - \vec{y} \|$. The vectors $\vec{x}$ and $\vec{y}$ are thus at least as close as $\Theta\vec{x}$ and $\Theta\vec{y}$. For

$$\vec{x}_{n+1} = \Theta\vec{x}_n, \qquad (1)$$

let the fixed point of convergence, if it exists, be denoted by $\vec{x}_\infty = \lim_{n\to\infty} \vec{x}_n$. When Θ is contractive, there is a

[2] A set, Φ, is said to be convex if $\vec{x} \in \Phi$ and $\vec{y} \in \Phi \to \alpha\vec{x} + (1-\alpha)\vec{y} \in \Phi$ for all $0 \le \alpha \le 1$.

[3] The linear solution of POCS outlined in the previous section can be couched as simultaneous solution of a set of linear equations [Shum *et al.*]. This is not the case for more general cases of POCS.

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unique fixed point that is independent of the initialization.[4]
This is the unique solution to [14]

$$\vec{x}_\infty = \Theta \vec{x}_\infty \tag{2}$$

For nonexpansive operations, on the other hand, the iteration in Equation (1) depends on the point of initialization of the iteration and can converge to a plurality of fixed points. Equation (2) thus has numerous solutions.

The projection operators Θ=**P** and Θ=***R*** illustrated in Figure 4 are examples of nonexpansive operators. The composite operator, Θ=***R*P**, is likewise nonexpansive. POCS, as illustrated in Figure 4, corresponds to application of Equation (1) for Θ=***RP***. If the manifolds in Figure 4 meet at a single point, convergence will be unique despite the fact that the composite operation is nonexpansive.

In Figure 6, the operator of one iteration is

$$\Theta \vec{x} = \boldsymbol{R} \vec{f}(\vec{x}). \tag{3}$$

Since the encoder has been trained to generate an identity operation for all sensor training data, it has a plurality of fixed points and Equation (2) does not have a unique solution. Therefore, $\vec{f}(\vec{x})$ cannot generally be contractive. It can, however, be nonexpansive. Since ***R*** is nonexpansive, the composite operation, Θ, defined by Equation (3), is nonexpansive if $\vec{f}(\vec{x})$ is a nonexpansive operator. Once trained, determination of the status of $\vec{f}(\vec{x})$ as being nonexpansive is important to the stability of iterative restoration of the failed sensors readings.

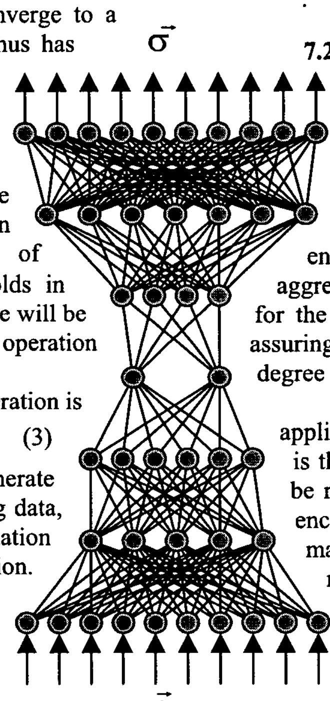

Figure 7: A multiplayer perceptron with a plurality of hidden layers is trained to form the $\vec{\sigma} = \vec{f}(\vec{s})$ mapping- an identity mapping for the training data.

We here give an outline on determining whether $\vec{f}(\vec{x})$ might be contractive or nonexpansive in the case where the constraint relationship is learned in a neural network of the type illustrated in Figure 7. If there are L layers (the input is not counted as a layer), the mapping can be written as

$$\vec{\sigma} = \vec{f}(\vec{s}) = \boldsymbol{S}_L \mathbf{W}_L \ldots \boldsymbol{S}_\ell \mathbf{W}_\ell \ldots \boldsymbol{S}_2 \mathbf{W}_2 \boldsymbol{S}_1 \mathbf{W}_1 \vec{s} \tag{4}$$

where $\mathbf{W}_\ell$ denotes the matrix of weights to the ℓth layer and $\boldsymbol{S}_\ell$ is a vector operator that subjects each element of an input vector to a sigmoid nonlinearity. If either of the conventional nonlinearities, tanh($\cdot$) or $[1+\exp(-(\cdot))]^{-1}$, are used, the operator, $\boldsymbol{S}_\ell$, is nonexpansive. The eigenvalues of the weight matrices will reveal them as nonexpansive. Imposition of a nonexpansive disposition through weight control in training [36] is a viable goal towards establishing the ability of this network to perform MISED restoration.

[4] A simple contractive operator example for positive argument in $\Re^1$ is $\Theta x = e^{-x}$. For all initializations, the iteration in (1) converges to x_∞=0.5671439...

7.2. Finding Missing Readings Using Search Techniques:

Missing readings from failed sensors can also be achieved through application of search techniques. A generalized approach is illustrated in Figure 8. Using known sensor readings as a subset of the input, the input and output of the encoder are examined. The difference is aggregated to a single error measurement. A search for the missing inputs commences with the goal of assuring the input and output match to a prespecified degree of precision.

There are several search algorithms applicable to this problem [36]. Of greater interest is the manner that the training of the encoder can be related to this search. In POCS, for example, encoder training resulting in a nonexpansive mapping is important for the subsequent POCS restoration phase. We anticipate similar relationships between the encoder training and the missing sample search will be important in the approach illustrated in Figure 8. Care must be taken, for example, to train the encoder so that false minima are avoided.

8. CONCLUSIONS

Sensor readings in a restricted environment can have readings relating to each other in such a matter that missing readings from a set of failed sensors can be restored from the readings of those remaining. When supplied as in input to a process or other control system, operation will still be possible. Overall performance will degrade gracefully.

Discovery of data constraints can be achieved by training an auto-encoder. The trained encoder can then be used to restore lost readings from failed sensors. This is achieved through either application of a POCS type algorithm or a more conventional search.

Our preliminary study suggests that solid theory exists for the case where data are linearly related. More powerful performance may exist for nonlinear relations.

9. ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation and The Boeing Phantom Works at The Boeing Company.

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