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SPACE-VARIANT HOLOGRAPHIC OPTICAL SYSTEMS USING PHASE-CODED REFERENCE BEAMS

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Abstract

A new holographic implementation of a sampling technique permits straightforward representations of two dimensional space-variant optical systems. The set of sample transfer functions required for the representation is sequentially recorded on a single holographic plate by utilizing a diffuser to produce phase-coded reference beams. The phase coding operation then acts to suppress crosstalk between the stored holograms when they are played back simultaneously. Because this approach does not depend on volume effects in the recording medium in an essential way, the holograms can either be produced digitally or optically. Basic concepts and the results of experimental investigations are presented and discussed.

Introduction

A procedure has recently been described for using a sampling technique to represent space variant optical systems.[1] One proposed implementation of this technique relies on volume hologram effects to angle-multiplex various sample transfer functions of the system into a thick recording medium.[2-4] A problem arises with angle multiplexing, however, in that the extinction angle effect is, for practical purposes, one-dimensional in nature due to the resulting "Bragg cones".[3,5] Thus, using volume holograms, only one-dimensional space-variant systems can be represented in a straightforward way, without undesirable crosstalk upon playback. More straightforward techniques have been demonstrated elsewhere for one-dimensional space-variant systems representation.[6-7]

This paper demonstrates a different holographic implementation technique for sampling theorem-based representations of two-dimensional space-variant systems. This implementation utilizes reference beam encoding of the various transfer functions via a random (or pseudo-random) phase diffuser. This approach eliminates, in principle, the need for a volume recording medium, so that thin (rather than thick) recording media may be used. It therefore opens the important possibility of using computer-generated holograms to simulate arbitrary two-dimensional space-variant operations not possible with volume hologram representations.

Phase encoded reference beams have been used previously for color holography,[8-9] and for the purpose of multiplexing a number of point source objects for digital storage.[10] The requirements in holographically representing space-variant systems are more stringent than for these applications in that simultaneous phase-coherent reconstruction of a number of recordings is desired with no significant objectionable crosstalk between the various playback (reference) beams. As we will show below, the random phase diffuser technique can satisfy these requirements.

Recording and Playback Operations With Phase-Coded Reference Beams

The space-variant system recording procedure based on the sampling theorem, but without reference beam phase encoding, is shown in Fig. 1.[2-3] The system S for the moment is shown sampled by impulse functions (point sources) i_1 to i_N, giving rise to the point spread functions h_1 to h_N at the output plane of S. The lens L_1 then produces fields proportional to the transfer functions H_1 to H_N at the hologram plane, where H_j is the Fourier transform of h_j.[11] Now for each input point source i_j there is a corresponding reference point source r_j which is transformed by the lens L_2 to form the reference plane wave R_j at the hologram plane.[2-3] The recording technique then consists of sequentially recording the interference patterns of the system function/reference plane wave pairs (H_j, R_j). If we bias the medium so that the resulting holograms's amplitude transmittance function, t, is proportional to the exposing intensity pattern, then

$$t = \sum_{j=1}^{N} |H_j + R_j|^2 \qquad (1)$$

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The playback step is shown in Fig. 2. The input object is spatially sampled at the point source locations of the original reference array to produce sampled inputs $s_1 r_1$ to $s_N r_N$, where s_j is the sampled value at the j-th location. The desired output plane field is given by the sum

$$o = \sum_{j=1}^{N} s_j \, F^{-1}\{H_h\}$$

$$= \sum_{j=1}^{N} s_j \, h_j \qquad (2)$$

where we note that coherent addition of the simultaneous reconstructions is required. Also note that potential crosstalk terms have been neglected,[12] and that low pass filtering in the hologram plane will be required to obtain a continuous output.[1]

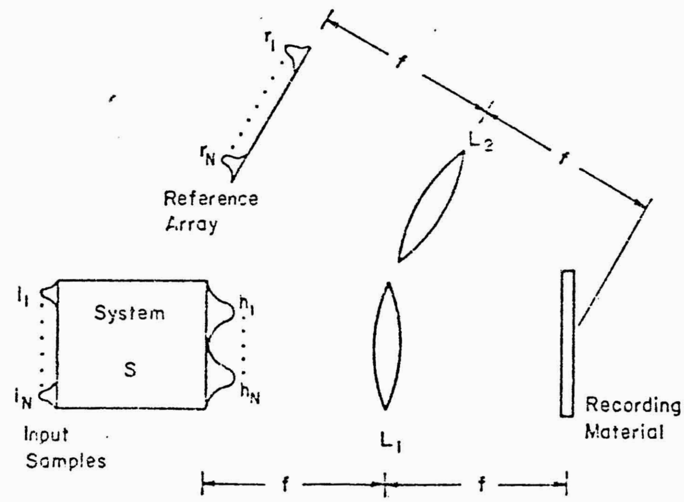

Fig. 1: Recoding scheme for the holographic representation of the space-variant system S.

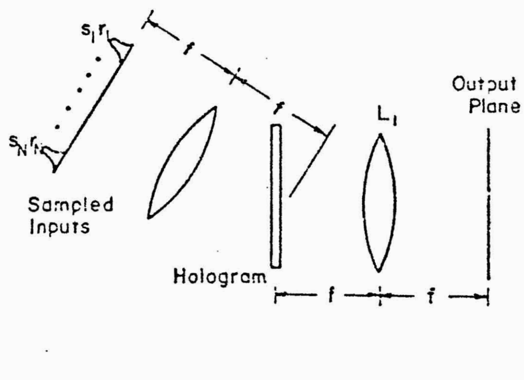

Fig. 2: Playback scheme for the holographic representation of the system S.

Since crossbalk terms have been neglected in writing Eq. (2), we have assumed that each input point sample "sees" only the desired transfer function upon playback. We will now analyze what happens in the special case of a simple two sample point system to show what the crosstalk problems are and to examine how utilization of the diffuser encoding technique in the reference beams can suppress crosstalk.

After sequential recording of the two object-reference pairs (H_1, R_1) and (H_2, R_2), the amplitude transmittance function stored in the hologram is given by

$$t = |H_1 + R_1|^2 + |H_2 + R_2|^2 \qquad (3)$$

In the playback step, if the sampled input values are given by the constants s_1 and s_2, the reconstructed wavefront just to the right of the hologram in Fig. 2 consists of 16 terms, four of which have been diffracted by the hologram to appear in the output plane. We will define these four terms as W', so that

$$W' \triangleq s_1 R_1 R_1^* H_1 + s_2 R_2 R_2^* H_2 + s_1 R_1 R_2^* H_2 + s_2 R_1^* R_2 H_1, \qquad (4)$$

where the superscript "*" denotes a complex conjugate.

Now suppose an ideal phase diffuser is placed in the reference beam side of the system as shown in Fig. 3, such that R_1 and R_2 are unit amplitude waves with complex phase fronts (i.e. $R_1 = \exp[j\phi_1(x,y)]$, $R_2 = \exp[j\phi_2(x,y)]$). We may take the inverse Fourier transform[12] of Eq. (4) to show that the output (image) plane field is given by

$$o' = F^{-1}\{W'\} = s_1 h_1 * [r_1 \star r_1] + s_2 h_2 * [r_2 \star r_2] + s_1 h_2 * [r_1 \star r_2] + s_2 h_1 * [r_2 \star r_1] \qquad (5)$$

where "*" represents convolution, and "$\star$" represents correlation. In obtaining Eq. (5) we have made use of the convolution and autocorrelation theorems of Fourier analysis.[11] Note that now, however, r_1 and r_2 represent the diffuser pattern seen at the two positions in the input sampling array. These two patterns are effectively multiplied by the sampled values s_1 and s_2, respectively. If the diffuser has the property that $r_i \star r_i$ is effectively a Dirac delta function, while $r_i \star r_j$ is a very broad, uniform spatial function, then the output may effectively be expressed as

$$o' = s_1 h_1 + s_2 h_2 + \text{diffuse background noise} \qquad (6)$$

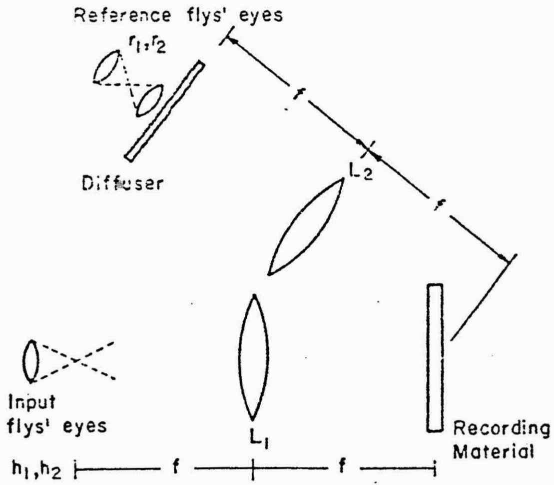

Fig. 3: System configuration for including phase diffusers in the reference beam to be used for both recording and playback.

It should be noted that no extinction angle was assumed operative in obtaining the result of Eq. (6). In this approach, we are thus not restricted to working with thick recording media,[2-3] a potentially significant advantage.

Experimental Results

To test the theory presented above, an idealized, space-variant two-point magnifier was set up as shown in Fig. 4. Two fly's-eye lenses (3.8 mm diameter each) having a vertical separation of 15.4 mm were used to form the reference point sources. The output of an idealized magnifier (with $M = 1/2$) was simulated by using two fly's-eye lenses separated by 7.7 mm to form the object points. The transforming lenses L_1 and L_2 had 10 cm focal lengths. The results of three experiments which were performed are described below.

Experiment 1: In the first experiment no diffuser was placed in the reference beams. The (R_1, H_1) and (R_2, H_2) combinations were recorded sequentially on a Kodak 649F plate. Reconstruction was performed by moving L_1 10cm. to the right of the hologram, and observing the image plane, which is now the Fourier transform plane of L_1 (see Fig. 2). The result is shown in Fig. 5 where the inner pair of points is the desired output (i.e. a pair of points separated by 7.7 mm.) and the outer pair of points represents the crosstalk terms

$F^{-1}[s_1H_2R_1R_2^*]$ and $F^{-1}[s_2H_1R_1^*R_2]$ which we want to suppress.

It should be noted here that the geometry of this experiment was such that the extinction angle effect was not operative in the vertical direction, so that the points in the vertical exhibit maximum cross-talk, as shown in Fig. 5.

Experiment 2: In this experiment, a shower glass diffuser was inserted in the reference beams, about 3 cm from the plane of the reference point sources so that each of the two beams would intercept a maximum diffuser area, but would not intersect the diffuser area intercepted by the other reference beam. The output plane result for this experiment is shown in Fig. 6. Here the two desired output points stand out clearly against a diffuse background of crosstalk noise. The design pattern induced by the particular shower glass diffuser used did not have a very sharp autocorrelation function, so that repositioning of the developed hologram was not too critical an operation (i.e. it took a movement of several millimeters to make the output points disappear). Note that the diffuser also had a spatially varying attenuation, so that the cross-talk is not spread as uniformly as possible over the output field. Nevertheless the results appear to be encouraging.

Experiment 3: A third experiment was performed to demonstrate the two-dimensional capabilities of the diffuser encoding technique. A three-point demagnifier was simulated with the same basic configuration as in Fig. 4, however three of the four lenses in a square subarray portion of the fly's-eye array were used. The result, when the diffuser was placed in the reference beams, is shown in Fig. 7. It can be seen that the multiple crosstalk points which would be expected without the diffuser encoding of the reference beams have been relegated to a diffuse background noise, while the three desired output points stand out clearly. Again it should be noted that since 649F has a rather large extinction angle relative to the angular separation of the reference beams used here, volume hologram effects were not responsible for eliminating either the horizontal or vertical cross-talk expected without the diffuser.

Conclusions

A technique has been demonstrated for holographically recording representations of two-dimensional space-variant systems using phase-encoded reference beams. The use of reference beam phase encoding for the purpose of multiplexing noninterfering holograms onto a single "thin" recording medium, rather than attempting to use the extinction angle property of a thick recording medium to angle multiplex the multiple holograms, possesses a number of potential advantages. One would not be restricted to volume holograms for recording space-variant optical systems. Thus one could in principle construct computer-generated holograms for representing arbitrary two-dimensional space-variant processors.

It is clear that the properties of the phase-coded diffusers are important in determining the practical limitations of the recording process. A number of diffuser studies have been made concerning the use of diffusers as spectrum-levelers for recording Fourier transforms of input data and as devices for speckle reduction. Several algorithms have resulted which give rise to phase masks with flat spectra and delta-like autocorrelations. In the space-variant processor, we would also like diffusers with these properties. In addition, we are dealing with aperiodic functions instead of periodic; we require a family of diffusers for any given mask size; and we need all cross-correlations between members of a diffuser family to be uniformly small so all holograms can be accessed simultaneously with minimal cross-talk. Using results from coding theory, two-dimensional bi-phase codes are being developed which have the desired properties. Additional investigations are underway in these and other areas relating to the technique presented in this paper.

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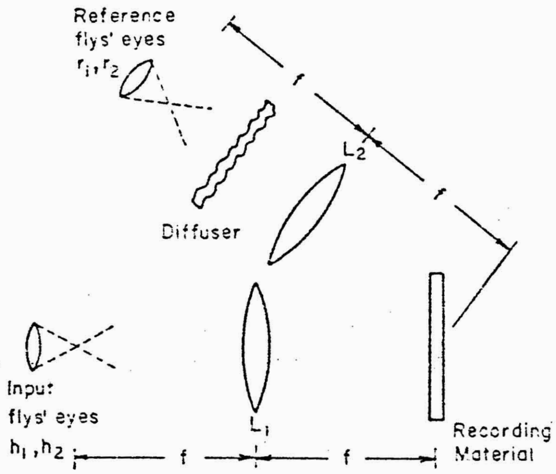

Fig. 4: Experimental setup for simulation of two-point magnifier.

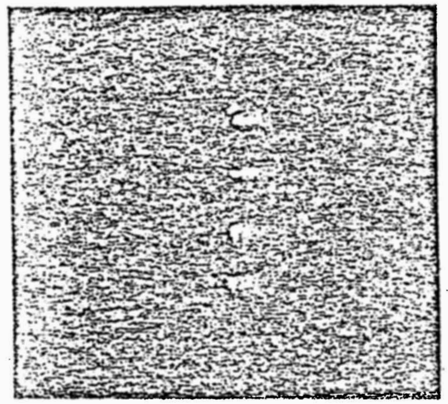

Fig. 5: Reconstructed output obtained with simulated two-point demagnifier and without diffuser-induced phase coding of reference beams. The two inner points are the desired output, while the two outer points represent crosstalk.

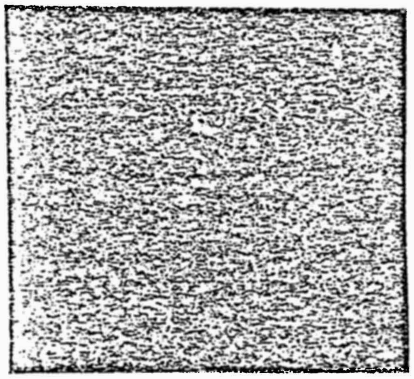

Fig. 6: Reconstructed output (Exp. 2) with phase coding of reference beams of shower glass diffuser. The crosstalk terms present in Fig. 4 have been spread out into diffuse background noise.

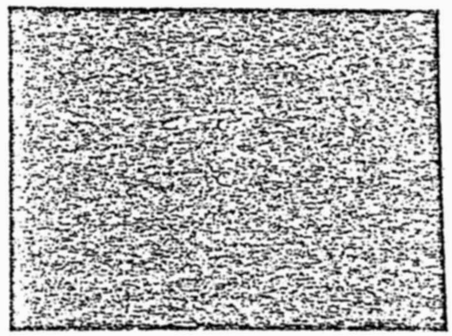

Fig. 7: Reconstructed output (Exp. 3) for the simulated three-point demagnifier, indicating diffuse background noise in both directions.

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