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COHERENT OPTICAL PROCESSORS FOR AMBIGUITY FUNCTION DISPLAY AND ONE-DIMENSIONAL CORRELATION/CONVOLUTION OPERATIONS

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Abstract

This paper presents a tutorial overview of some recently developed coherent optical processing techniques for performing certain types of computations often encountered in radar signal processing applications. The processors described utilize two one-dimensional transparencies which represent the spatial analogs of temporal waveforms. These transparencies are commonly rotated 45° in opposite directions, and serve as the inputs to a coherent optical processor which performs a one-dimensional Fourier transform. For various orientations of the input transparencies, possible operations include a simultaneous range/Doppler display of the input's ambiguity function (for identical input transparencies) and a cross-ambiguity function display (for non-identical transparencies). One dimensional coherent correlations and convolutions can similarly be performed without the motion or Fourier transform encoding commonly required in conventional coherent processors. Sample experimental results are presented along with the basic theory.

Introduction

Recent developments in coherent optical processing make possible real time display of a wide class of one-dimensional signal operations.(1-3) In this paper, a tutorial overview of such schemes is presented with emphasis placed on those operations often encountered in radar signal processing. These include ambiguity function display, correlation, and convolution type operations.

Ambiguity Function Display

The ambiguity function, first introduced by Woodward,(4) has been applied in radar in predicting the capability of a given signal to determine simultaneously the range and velocity of a target. The range is determined by the time delay τ and the velocity by the Doppler shift ν. The ambiguity function for a given real-valued signal f(t) is

$$\chi(\nu,\tau) = \int_{-\infty}^{\infty} f(t)f(t-\tau)\exp(-j2\pi\nu t)\,dt, \tag{1}$$

In optics, Papoulis has employed the ambiguity function in analyzing diffraction phenomena.(5)

Cutrona et al.(6,7) and Preston(8) have proposed a coherent ambiguity function processor which utilizes multiple channels to display the ambiguity function for discrete values of τ. The scheme of Casasent et al.(9) generates 1-D slices of the ambiguity function in the (ν,τ) plane. Similar 1-D displays have also been electronically produced.(10) The method we describe(11-12) (1) displays $|\chi(\nu,\tau)|^2$ in a continuous (rather than quantized) form over the entire (ν,τ) plane, (2) has the capacity for extension to real time processing, and (3) is easily implemented.

Geometrical Interpretation

On the (t,τ) plane, a function f(t) takes on the 1-D nature exemplified in Fig. 1(a). Upon rotating this function counterclockwise about the origin through an angle θ, we generate the function [Fig. 1(b)]

$$f(t\cos\theta + \tau\sin\theta). \tag{2}$$

Thus, for a rotation of 45°, we obtain $f[(t+\tau)/\sqrt{2}]$, and for a rotation of -45°, we obtain $f[(t-\tau)/\sqrt{2}]$. Consider, then, multiplying these two functions [Fig. 1(c)] and performing a Fourier transformation with respect to t:

$$\int_{-\infty}^{\infty} f\left(\frac{t+\tau}{\sqrt{2}}\right) f\left(\frac{t-\tau}{\sqrt{2}}\right) \exp(-j2\pi\nu t)\,dt, \tag{3}$$

where ν is the frequency variable associated with t. Upon making the variable substitution $t' = (t + \tau)/\sqrt{2}$, Eq. (3) becomes a scaled version of the ambiguity function of Eq. (1):

$$\sqrt{2}\exp(j2\pi\nu\tau) \int_{-\infty}^{\infty} f(t')f(t' - \sqrt{2}\tau)\exp[-j2\pi(\sqrt{2}\nu)t']dt' = \sqrt{2}\chi(\sqrt{2}\nu,\sqrt{2}\tau)\exp(j2\pi\nu\tau). \tag{4}$$

Thus, apart from a multiplicative phase term, we may generate a scaled version of the ambiguity function by representing the time delay by simple 45° rotations and the Doppler shift by an appropriate 1-D Fourier transformation.

The ν and τ axes of the ambiguity function can be scaled by choosing another appropriate value of θ rather than 45°. If two identical versions of f(t) are each rotated angles of θ and $-\theta$ and then are Fourier transformed, the result is

$$\int_{-\infty}^{\infty} f(\alpha t + \beta\tau)f(\alpha t - \beta\tau)\exp(-j2\pi\nu t)\,dt$$

$$= \frac{1}{\alpha}\chi\left(\frac{\nu}{\alpha}, 2\beta\tau\right)\exp(-j2\pi\beta\tau\nu/\alpha)$$

where $\alpha = \cos\theta$ and $\beta = \sin\theta$. Scaling can thus be performed simply by altering θ. For clarity of presentation, however, we will henceforth restrict attention to the case where $\theta = 45^{\circ}$.

Implementation Scheme

A processor capable of performing a 1-D Fourier transform is shown in Fig. 2. When plane Pl is illuminated from the left with a coherent plane wave of wavelength λ, then the field amplitude $U(\nu,\tau)$ is plane P_2 is related to the transmittance $s(t,\tau)$ in plane P_1 by

$$U(\nu,\tau) = \exp(-j2\pi\lambda f\nu^2) \int_{-\infty}^{\infty} s(t,-\tau)\exp(-j2\pi\nu t)\,dt, \tag{5}$$

where f is the focal length of both lenses L_1 and L_2,[13] and the spatial frequency ν is related to the horizontal displacement x_2 on plane P_2 by

$$\nu = x_2/\lambda f. \tag{6}$$

Consider, then, placing two identical 1-D transparencies of f(t) in plane P_1, each rotated 45° in such a manner as to form the product

$$s(t,\tau) = f\left(\frac{t+\tau}{\sqrt{2}}\right) f\left(\frac{t-\tau}{\sqrt{2}}\right). \tag{7}$$

The corresponding field amplitude in plane P_2 [from Eq. (5)] is then given by

$$U(\nu,\tau) = \exp(-j2\lambda f\nu^2) \int_{-\infty}^{\infty} f\left(\frac{t-\tau}{\sqrt{2}}\right) f\left(\frac{t+\tau}{\sqrt{2}}\right)\exp(-j2\pi\nu t)\,dt$$

$$= \sqrt{2}\exp[-j2\pi\nu(\tau+\lambda f\nu)]$$

$$\times \int_{-\infty}^{\infty} f(t')f(t' - \sqrt{2}\tau)\exp[-j2\pi(\sqrt{2}\nu)t']dt', \tag{8}$$

where, as before, we have made the change of variable $t' = (t+\tau)/\sqrt{2}$. The intensity distribution associated with Eq. (8) is immediately recognized as a scaled version of the squared modulus of the ambiguity function:

$$\begin{aligned} I(\nu,\tau) &= |U(\nu,\tau)|^2 \\ &= 2|\chi(\sqrt{2}\nu,\sqrt{2}\tau)|^2. \end{aligned} \tag{9}$$

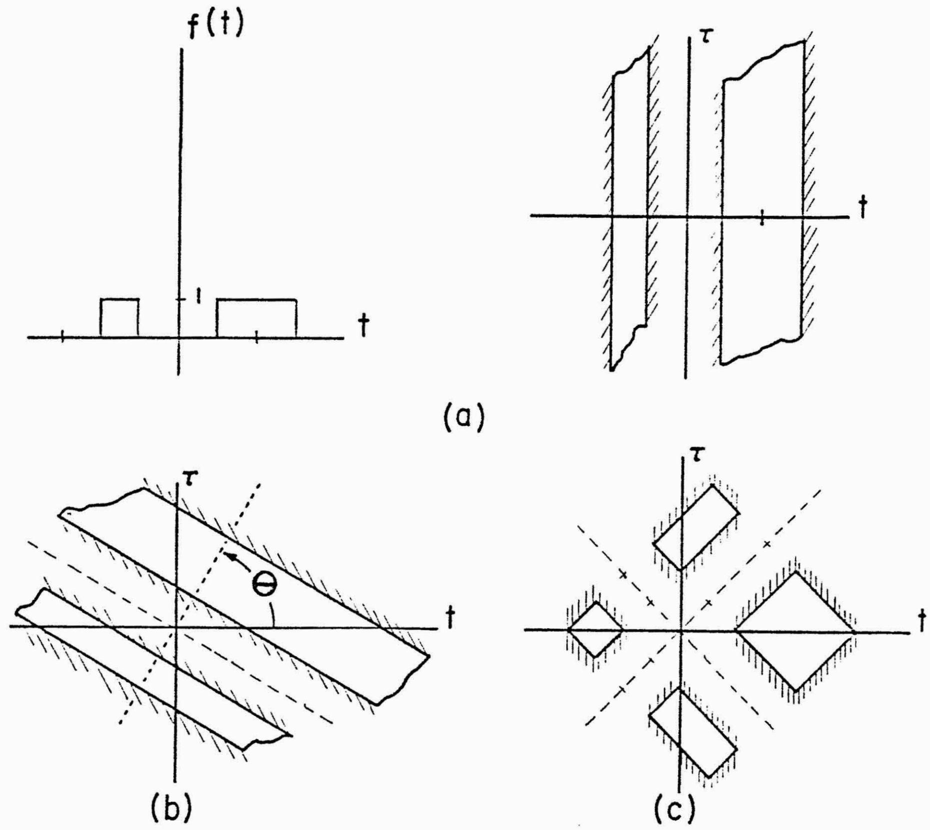

Fig. 1. (a) A function f(t) in time and in the (t,τ) plane. (b) By rotating f(t) counterclockwise an angle of θ about the origin of the (t,τ) plane, we generate f(t cosθ + τ sinθ). (c) The function f[(t+τ)/√2]f[(t−τ)/√2] in the (t,τ) plane.

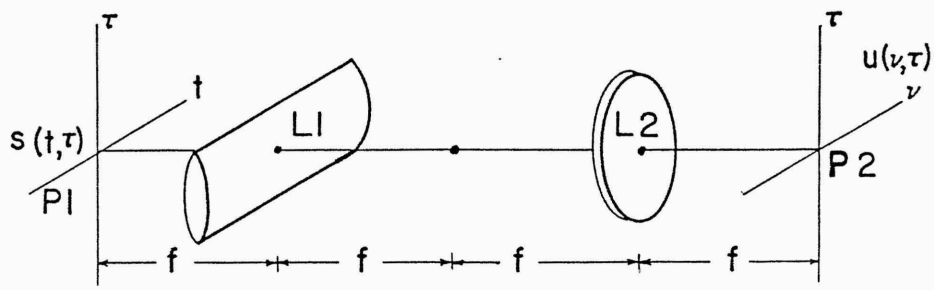

Fig. 2. A coherent processor for ambiguity function diaplay. Both the lenses have focal length f. Fourier transformation is performed in the horizontal direction and imaging in the vertical direction.

In practice, the two one dimensional inputs can be generated by two appropriately rotated 1D electro-optical transducers. Said and Cooper have implemented this scheme using ultrasonic light modulators for real time applications.[12]

Experimental Results

To evaluate the performance of the proposed processor, the ambiguity functions for a single and double pulse signal are evaluated analytically and compared to the corresponding optical system outputs. In practice, the processor output is magnified by conventional means for observation and photographic purposes.

Single Pulse: For a single pulse we may write

$$f(t) = \mathrm{rect}(t/2T), \tag{10}$$

where 2T is the pulse duration, and

$$\mathrm{rect}(t) \triangleq \begin{cases} 1; & |t| \leq 1/2, \\ 0; & |t| > 1/2. \end{cases}$$

Substituting Eq. (10) into Eq. (1) followed by evaluation yields the ambiguity function

$$\chi(\nu,\tau) = \begin{cases} (2T - |\tau|)\,\mathrm{sinc}\,\nu(2T - |\tau|)\exp(-j\pi\nu\tau); & |\tau| \leq 2T \\ 0; & |\tau| \geq 2T, \end{cases} \tag{11}$$

where

$$\mathrm{sinc}\,\nu \triangleq (\sin\pi\nu)/\pi\nu.$$

The corresponding output intensity is

$$|\chi(\nu,\tau)|^2 = \begin{cases} (2T - |\tau|)^2\,\mathrm{sinc}^2\nu(2T - |\tau|); & |\tau| \leq 2T \\ 0; & |\tau| \geq 2T. \end{cases} \tag{12}$$

For purposes of identification, it is instructive to examine the locus of points where the ambiguity function is identically zero. From Eq. (12), this zero locus may easily be shown to be

$$\nu = n/(2T - |\tau|); \quad |\tau| \leq 2T, \tag{13}$$

where n is any nonzero integer. The piecewise hyperbolic nature of these curves is shown in Fig. 3.

To generate the ambiguity function for a single pulse, two identical slits of width 2T are rotated 45° and -45° on the (t,τ) plane to form the product

$$f\left(\frac{t-\tau}{\sqrt{2}}\right) f\left(\frac{t+\tau}{\sqrt{2}}\right)$$

This product is recognized as a 2T by 2T square with diagonals along the t and τ axis. This "diamond" is then placed in plane P_1 of the coherent optical processor of Fig. 2. The result is shown in Fig. 4. As can be seen, the coherent processor output compares quite nicely with the theoretical result in Fig. 3. A 3-D computer graph of the corresponding ambiguity function modulus may be found in Fig. 6.6 of Rihaczek.[14]

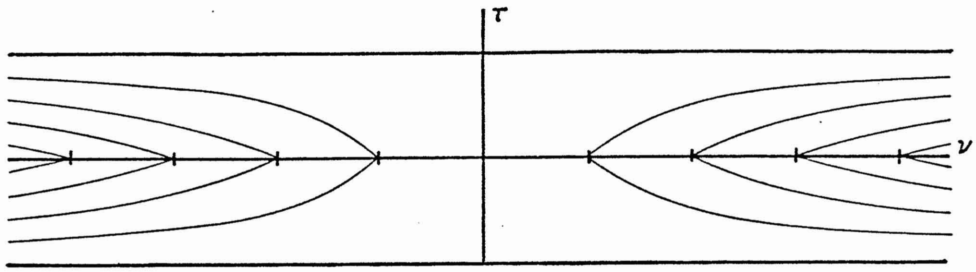

Fig. 3. Zero locus plot of the ambiguity function of a single pulse.

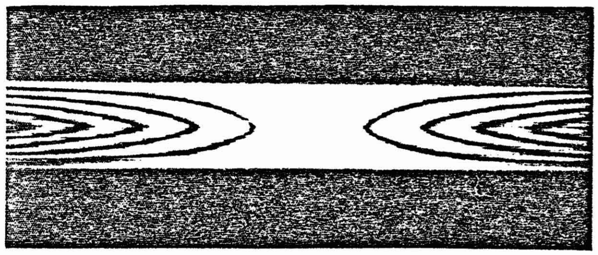

Fig. 4. The ambiguity function (modulus squared) display for a single pulse, as generated by the coherent processor of Fig. 2.

<u>Double Pulse</u>: For a double pulse [Fig. 5(a)], we write

$$f(t) = \mathrm{rect}[(t + 2T)/2T] + \mathrm{rect}[(t - 2T)/2T], \tag{14}$$

where, for convenience, the pulse separation 2T has been chosen to be equal to each pulse width. The geometrical interpretation of $f[(t - \tau)/\sqrt{2}]f[(t + \tau)/\sqrt{2}]$ is shown in Fig. 5(b). The corresponding squared modulus of the ambiguity function is

$$|\chi(\nu,\tau)|^2 = \begin{cases} 4(2T - |\tau|)^2 \mathrm{sinc}^2 \nu(2T - |\tau|)\cos^2(4\pi T\nu); & |\tau| \le 2T \\ (2T - |\tau|)^2 \mathrm{sinc}^2 \nu(2T - |\tau|); & 2T \le |\tau| \le 4T \\ (6T - |\tau|)^2 \mathrm{sinc}^2 \nu(6T - |\tau|); & 4T \le |\tau| \le 6T \\ 0 \quad ; & |\tau| \ge 6T. \end{cases} \tag{15}$$

The equations describing the zero-value loci are easily shown to be

$$\begin{aligned} \nu &= (2m + 1)/8T; |\tau| \le 2T, \\ \nu &= n/(|\tau| - 2T); |\tau| \le 4T, \\ \nu &= n/(6T - |\tau|); 4T \le |\tau| \le 6T, \end{aligned} \tag{16}$$

where m is any integer, and, as before, n is any nonzero integer. An illustration of these zero-value loci is offered in Fig. 6.

By placing the transparency of Fig. 5(b) in plane P_1 of the coherent processor, the ambiguity function for the double pulse is generated. The result, shown in Fig. 7, again compares quite favorably with the theory.

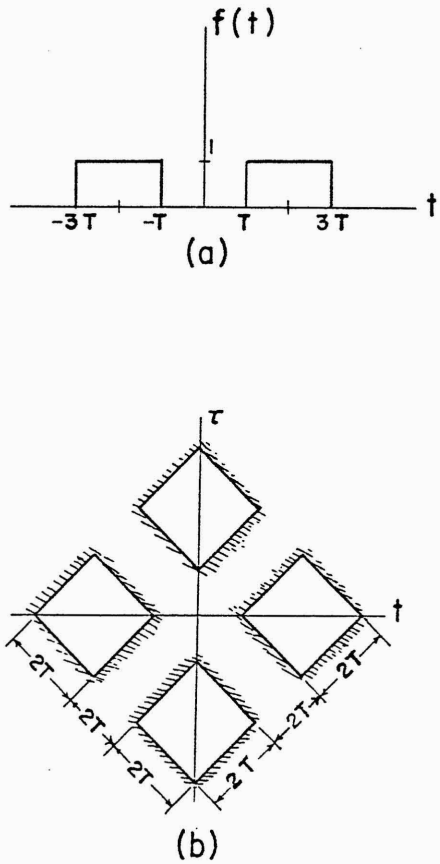

Fig. 5. (a) A double pulse. (b) The corresponding function $f[(t + \tau)/\sqrt{2}]f[(t - \tau)/\sqrt{2}]$ in the (t,τ) plane.

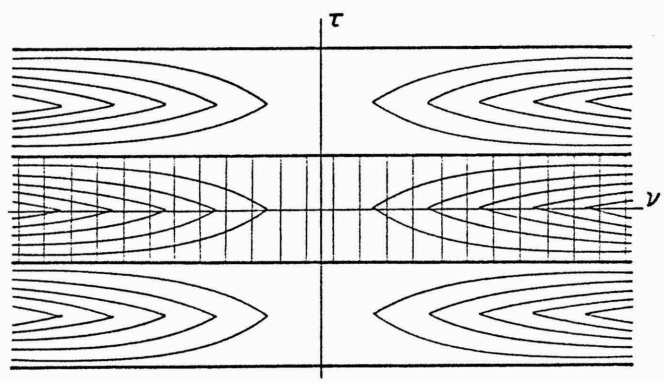

Fig. 6. Zero locus plot of the ambiguity function of a double pulse.

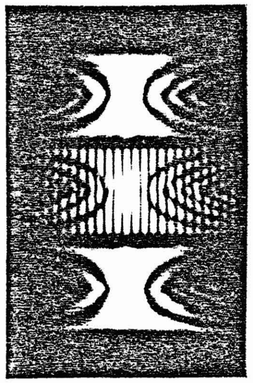

Fig. 7. The ambiguity function (modulus squared) display for a double pulse as generated by the coherent processor of Fig. 2.

Cross-Ambiguity Function Display

In a manner identical to that of the ambiguity function, the cross ambiguity function $\chi_c(\nu;\tau)$ of two signals, f(t) and g(t), can be generated by the coherent processor of Fig. 2, where

$$\chi_c(\nu;\tau) \triangleq \int_{-\infty}^{\infty} f(t)g(t-\tau)\exp(-j2\pi\nu t)dt \tag{17}$$

Here, two 1-D transparencies representing $f(\cdot)$ and $g(\cdot)$ are appropriately rotated in the processor input plane and the corresponding cross-ambiguity function will appear in the processor's output plane.

Correlation and Convolution

The capability of conventional coherent processors to perform two-dimensional correlation and convolution operations is well known. All these schemes, however, either necessitate encoding a Fourier transform on a transparency or require motion. Using the ambiguity function processor, 1-D convolution and correlation can be performed with no requirement of motion or Fourier encoding, thus offering a scheme whereby such operations can be performed in real time.

The autocorrelation of a real valued signal $f(\cdot)$ is defined by

$$f(\tau) \star f(\tau) = \int_{-\infty}^{\infty} f(t)f(t-\tau)dt \tag{18}$$

Comparison with the ambiguity function expression in Eq. 1 reveals that

$$\chi(0,\tau) = f(\tau) \star f(\tau) \tag{19}$$

That is, the autocorrelation of $f(\cdot)$ appears along the τ axis of the ambiguity function. Thus, the ambiguity function processor can also be viewed as a one dimensional correlator. For 45° rotations of input transparencies, the output along the τ axis (i.e., $\nu = 0$) in the processor output plane of Fig. 2 is [from Eq. 8]:

$$U(0,\tau) = \sqrt{2}\int_{-\infty}^{\infty} f(t)f(t-\sqrt{2}\,\tau)dt \tag{20}$$

which is a scaled version of our desired result.

In a similar fashion, we can perform the crosscorrelation operation: